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NEW TESTS OF PERTURBATIVE QCD INSPIRED BY HYPOTHETICAL TAU LEPTONS [a]

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Inspired by the relation between the hadronic decay of the τ lepton and the e^+e^- annihilation into hadrons, we derive new tests of perturbative QCD. We design a set of commensurate scale relations to test the self-consistency of leading-twist QCD predictions for any observable which defines an effective charge. This method provides renormalization scheme and scale invariant probes of QCD which can be applied over wide data ranges.

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1 Introduction

The τ lepton hadronic width, $R_\tau = \Gamma(\tau^- \to \nu_\tau + \text{hadrons})/\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)$ plays an important role in the determination of the QCD coupling [1]. Its analysis has been performed using integral moments which minimize the sensitivity to the low energy data [2]. In particular, just by integrating the measured spectral functions up M we can simulate the physics of hypothetical τ leptons[2] with masses M smaller than the physical one. Their hadronic widths yield a crucial test of perturbative QCD (PQCD), since they are related to the e^+e^- annihilation cross section into hadrons $R_{e^+e^-}$ through

$$R_\tau(M) = \frac{2}{(\sum_f q_f^2)} \times \int_0^{M^2} \frac{d\,s}{M^2}\left(1-\frac{s}{M^2}\right)^2\left(1+\frac{2s}{M^2}\right) R_{e^+e^-}(\sqrt{s}). \qquad (1)$$

In this paper we report on a recent proposal [3] of self-consistency tests of PQCD, motivated by the above relations, which can be applied to any observable which defines an effective charge. These tests are examples of relations between observables at two different scales, which are called "commensurate scale relations" [4].

Effective charges are defined as the entire radiative contribution to an observable [5]. For instance, assuming f massless flavors, we have

$$R_{e^+e^-}(\sqrt{s}) \equiv (3\sum_f q_f^2)\left[1+\frac{\alpha_R(\sqrt{s})}{\pi}\right] \quad , \quad R_\tau(M) \equiv R_\tau^0(M)\left[1+\frac{\alpha_\tau(M)}{\pi}\right], \qquad (2)$$

where the effective charges α_R and α_τ can be written as a series in α_s/π in any given renormalization scheme. Their relevance is given by the fact that they satisfy the renormalization group equation with the same coefficients β_0 and β_1 as the usual coupling α_s.

At this point we can make use of the Mean Value Theorem in eq.(1), to relate α_R and α_τ by a scale shift

$$\alpha_\tau(M) = \alpha_R(\sqrt{s^*}), \quad \text{with} \quad \lambda_\tau = \frac{\sqrt{s^*}}{M} = \exp\left[-\frac{19}{24} - \frac{169}{128}\frac{\alpha_R(M)}{\pi} + \cdots\right], \qquad (3)$$

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where the value of λ_τ is a *prediction of NLO leading twist QCD*. This result was first obtained in [4] by using NNLO, however, we will see how it is due to the fact that both effective charges evolve with universal β_0 and β_1 coefficients.[3]

2 Tests of PQCD for a general observable

These relations can be generalized to arbitrary observables $O(s)$, with an associated effective charge α_O, by defining new effective charges

$$\alpha_f(M) \equiv \frac{\int_0^{M^2} \frac{ds}{M^2} f\left(\frac{s}{M^2}\right) \alpha_O(\sqrt{s})}{\int_0^{M^2} \frac{ds}{M^2} f\left(\frac{s}{M^2}\right)}, \tag{4}$$

where we can choose $f(x)$ to be any smooth, integrable function of $x = s/M^2$. Once more

$$\alpha_f(M) = \alpha_O(\sqrt{s_f^*}), \qquad\qquad 0 \le s_f^* \le M^2. \tag{5}$$

Note that this relation only involves data for the observable $O(s)$ and thus provides a self-consistency test for the applicability of leading twist QCD. To obtain the relation between the commensurate scales, we consider the running of α_O up to third order

$$\frac{\alpha_O(\sqrt{s})}{\pi} = \frac{\alpha_O(M)}{\pi} - \frac{\beta_0}{4}\ln\left(\frac{s}{M^2}\right)\left(\frac{\alpha_O(M)}{\pi}\right)^2 + \frac{1}{16}\left[\beta_0^2\ln^2\left(\frac{s}{M^2}\right) - \beta_1\ln\left(\frac{s}{M^2}\right)\right]\left(\frac{\alpha_O(M)}{\pi}\right)^3 \ldots \tag{6}$$

We substitute for α_O in eq. (4) to find

$$\frac{\alpha_f(M)}{\pi} = \frac{\alpha_O(M)}{\pi} - \frac{\beta_0}{4}\left(\frac{I_1}{I_0}\right)\left(\frac{\alpha_0(M)}{\pi}\right)^2 + \frac{1}{16}\left[\beta_0^2\left(\frac{I_2}{I_0}\right) - \beta_1\left(\frac{I_1}{I_0}\right)\right]\left(\frac{\alpha_O(M)}{\pi}\right)^3 \ldots, \tag{7}$$

where $I_l = \int_0^1 f(x)(\ln x)^l dx$. Hence

$$\lambda_f \equiv \frac{\sqrt{s_f^*}}{M} = \exp\left[\frac{I_1}{2I_0} + \frac{\beta_0}{8}\left(\left(\frac{I_1}{I_0}\right)^2 - \frac{I_2}{I_0}\right)\frac{\alpha_O(M)}{\pi}\right]. \tag{8}$$

Note that λ_f is constant to leading order, and therefore α_f satisfies the same renormalization group equation as α_O with the same coefficients β_0 and β_1; i.e., α_f is an effective charge.

Note that eq.(5) relates an observable with an integral over itself. It is also possible to obtain differential relations [6], but here we will simply illustrate the use of the integral relations.

3 Example: self-consistency test of $R_{e^+e^-}$ data.

Let us then set $O = R_{e^+e^-}$. In order to suppress the low energy region, where non-perturbative effects are important, we shall set $f(x) = x^k$, with k some positive number. Thus

$$\alpha_k(M) = \alpha_R(\lambda_k\, M) \quad \text{with} \quad \lambda_k = e^{\frac{-1}{2(1+k)}}, \tag{9}$$

When comparing with $R_{e^+e^-}$ data, we take into account the mass effects using [7]:

$$R_{e^+e^-}(\sqrt{s}) = 3\sum_1^f q_i^2 \frac{v_i(3-v_i^2)}{2}\left[1 + g(v_i)\frac{\alpha_R(\sqrt{s})}{\pi}\right] \equiv R_0(\sqrt{s}) + R_{Sch}(\sqrt{s})\frac{\alpha_R(\sqrt{s})}{\pi} \tag{10}$$

$$g(v) = \frac{4\pi}{3}\left[\frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right] \tag{11}$$

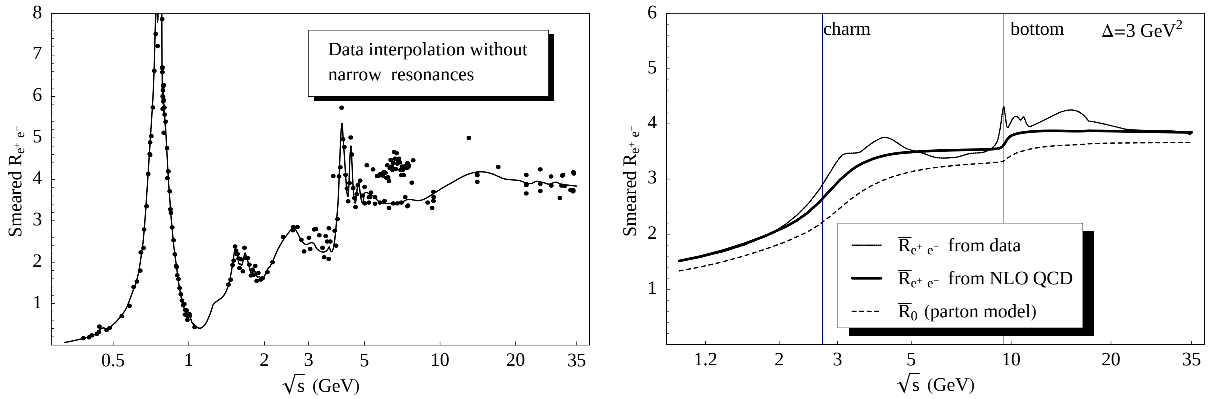

Figure 1.a) Interpolation of the central values of $R_{e^+e^-}$ data (see [3] for references). Note the discrepancy in the central values of experiments between 5 and 10 GeV. **b)** Smeared $R_{e^+e^-}$.

where v_i is the velocity of the initial quarks in their CM frame. The $v_i(3-v_i^2)/2$ factor is the parton model mass dependence and $g(v)$ is a QCD modification of the Schwinger correction. The quark masses have been taken as effective parameters which provide a good fit to the smeared data. All these corrections spoil eq.(9), but they are only important near the quark thresholds. At higher energies they tend to unity and quark masses become irrelevant, which is why our study is restricted to this regime. Still we cannot compare directly with the data since we observe hadrons, not quarks. Following [7] we define smeared quantities as follows:

$$\bar{R}(\sqrt{s}) = \frac{\Delta}{\pi}\int_0^\infty \frac{R(\sqrt{s'})}{(s-s')^2-\Delta^2}\, d\,s' \qquad (12)$$

By smearing $R_{e^+e^-}$ over a range of energy, ΔE, we focus the physics to the time $\Delta t = 1/\Delta E$ where an analysis in terms of quarks and gluons is appropriate. In what follows we use the standard value $\Delta = 3\,\mathrm{GeV}^2$ [7,8]. The smearing effect can be seen comparing Fig.1.a, which shows an interpolation of the $R_{e^+e^-}$ data, (see [3] for references) with Fig. 1.b. Note that *any fit using the QCD functional dependence will always satisfy* eq.(9) identically. To avoid this bias, we have parameterized the narrow resonances using their Breit-Wigner form, and we have interpolated the remaining data (see [3] for details).

Finally, using eqs.(10) and (12), we define smeared charges:

$$\bar{\alpha}_R(\sqrt{s}) = \frac{\bar{R}_{e^+e^-}(\sqrt{s}) - \bar{R}_0(\sqrt{s})}{\bar{R}_{Sch}(\sqrt{s})}, \qquad (13)$$

and similarly for $\bar{\alpha}_k$. According to the previous discussion we expect the smeared charges to satisfy eq.(8) in energy regions where the threshold corrections can be neglected.

Thus, in Fig.2.a we compare $\bar{\alpha}_R(\sqrt{s^*})$ with $\bar{\alpha}_k(\sqrt{s^*}/\lambda_k)$. The agreement for α_0 is poor since the low energy region is not suppressed enough. However we find a reasonable agreement for α_1 in several regions, agreement which disappears if we do not shift the scales. There are two regions of particular interest where we find a disagreement: First, from 5 to 10 GeV where there is a well known incompatibility between experiments (see Fig.1.a and ref.[9]). In Fig.1.a. we have kept the most recent data, as it is standard in the literature, but still their central values are systematically lower than the QCD predictions. which is why eq.(9) does not seem to hold. Our test correctly shows this incompatibility.

Second, we show in Fig. 2.b., the physical τ region where, considering that we are using LO QCD and central data values, the agreement looks quite satisfactory. This is encouraging for the applicability of PQCD in the region near the real τ lepton. However, at energies $\sqrt{s} \sim 1.5$ GeV, our results support the claims that the $R_{e^+e^-}$ data could be 6-7% lower than expected from R_τ data[10]. Note, however, that our conclusions have been obtained using *only data on* $R_{e^+e^-}$.

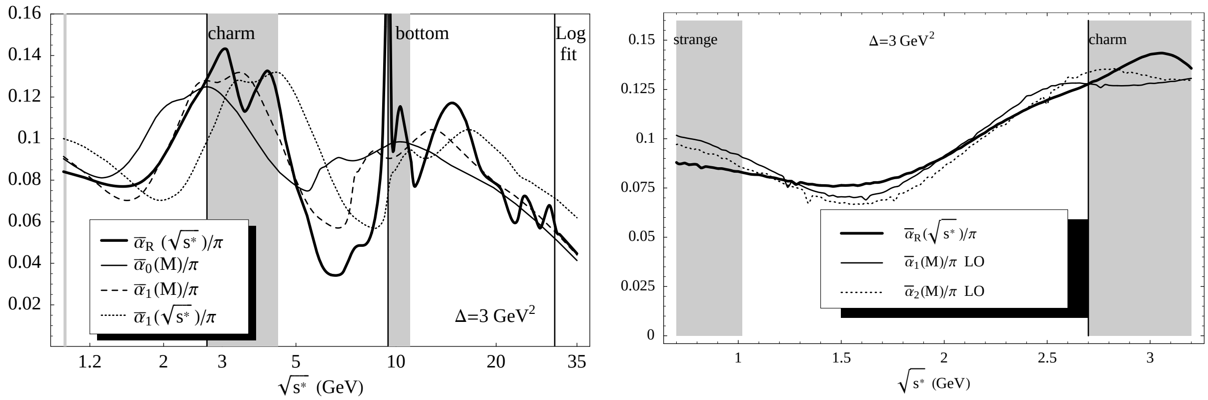

Figure 2.a) Comparison between $\bar{\alpha}_R(\sqrt{s^*})$ and different $\bar{\alpha}_k$ moments at $M = \sqrt{s^*}/\lambda_k$. The dotted line shows how the agreement is spoilt if we do not shift $\sqrt{s^*}$ to M. **b)** Comparison between $\bar{\alpha}_R(\sqrt{s^*})$ and different $\bar{\alpha}_k$ moments at $M = \sqrt{s^*}/\lambda_k$ in the low energy region.

4 Conclusions

Motivated by the relation between $R_{e^+e^-}$ and the R_τ, as well as the ideas of commensurate scale relations, we have presented new tests of PQCD. They can be applied in a wide energy range to any observable which defines an effective charge, and they are renormalization scheme and scale independent.

As an example, we have tested the self-consistency of existing $R_{e^+e^-}$ data according to PQCD. We have found a good agreement in the real τ region but incompatibilities around the 1.5 GeV region and in the range of 5 to 10 GeV, supporting previous claims obtained by different methods. The advantage of our approach is that it only relates the observable with itself, which can be very useful when applied to other experiments.

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References

1. E. Braaten, Phys. Rev. Lett. **60** (1988), Phys. Rev. **D39** 1458 (1989); E. Braaten, S. Narison and A. Pich, Nucl. Phys. **B373** 581 (1992).
2. F. Le Diberder and A. Pich, Phys. Lett. **B289** 165 (1992); S. Narison and A. Pich, Phys. Lett. **B304** 359 (1993), S. Narison, Phys. Lett. **B358** 113 (1995) and Phys. Lett. **B361** 121 (1995), M. Girone and M. Neubert, Phys. Rev. Lett. **76** 3061 (1996); S. Groote, J. G. Korner and A. A. Pivovarov, Phys. Lett. **B407** 66 (1997); M. Davier, hep-ph/9802447; R. Barate *et al.* Eur. Phys. J. **C4** 409 (1998). K. Ackerstaff *et al.*, hep-ex/9808019.
3. S. J. Brodsky, J. R. Peláez and N. Toumbas, slac-pub-7976, hep-ph/ 9810424. To appear in Phys.Rev.D.
4. S. J. Brodsky and H. J. Lu Phys. Rev. **D51** 3652 (1995).
5. G. Grunberg, Phys. Rev. **D29** 2315 (1984).
6. S. J. Brodsky, C. Merino and J. R. Peláez. slac-pub-8033.
7. E. C. Poggio, H. R. Quinn and S. Weinberg, Phys. Rev. **D13** 1958 (1976). T. W. Appelquist and H. D. Politzer, Phys. Rev. Lett. **34**, 43 (1975); Phys. Rev. **D12** 1404 (1975). J. Schwinger, *Particles, Sources and Fields*, Vol.II, Addison-Wesley, New York, 1973.
8. A. C. Mattingly and P.M. Stevenson, Phys. Rev. **D49** 437 (1994).
9. M. L. Swartz, Phys. Rev. **D53** 5268 (1996).
10. S. Groote *et al.*, Phys. Rev. Lett. **79** 2763 (1997).